

$$L(q', \dot{q}', t) = L(q + \delta q, \dot{q} + \delta \dot{q}, t) = L(q, \dot{q}, t) + \frac{\delta L(q, \dot{q}, t)}{\delta q} \delta q + \frac{\delta L(q, \dot{q}, t)}{\delta \dot{q}} \delta \dot{q}$$