

$$L'(T_t q', t) = L(T_t q', t) + \frac{d}{dt} V(T_t q', t)$$