

$$L(T_t q', t) = L(T_t q + T_t \delta q, t) = L(T_t q, t) + \int_{\mathbb{R}} \mathrm{d}\sigma \frac{\delta L(T_t q, t)}{\delta q(\sigma)} \delta q(\sigma)$$

$$= L(T_t q, t) + \int_{\mathbb{R}} \mathrm{d}\sigma \lambda(q, t, \sigma) \delta q(\sigma)$$