

$$L(T_t q) \coloneqq L(T_t \dot{q}_0, T_t q_0) \equiv L(\dot{q}(t), q(t))$$