

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{q}}\delta q\right]-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}}\delta q=\frac{\partial L}{\partial \dot{q}}\delta \dot{q}$$