

$$S(\tilde{q}, R) = \int_{|t| \leq R} \mathrm{d}t \left[\frac{1}{2} \dot{\tilde{q}}^2(t) - \frac{\omega^2}{2} \tilde{q}^2(t) + \frac{g}{4} \tilde{q}(t) \int_{\mathbb{R}} \mathrm{d}\zeta K(\zeta) \tilde{q}(t - \zeta) \right]$$