$$\int_{t_0'}^{t_1'} \mathrm{d}t\, L'(T_t\tilde{q}', t) = \int_{t_0}^{t_1} \mathrm{d}t \left\{ L'(T_t\tilde{q}', t) + \frac{\mathrm{d}}{\mathrm{d}t}\left[L(T_t\tilde{q}, t)\delta t\right] \right\}$$