

$$\begin{aligned} \lim_{R \rightarrow \infty} \delta S(q, R) &\equiv \lim_{R \rightarrow \infty} \int_{|t| \leq R} dt \int_{\mathbb{R}} d\sigma \frac{\delta L(T_t q, t)}{\delta q(\sigma)} \delta q(\sigma) \\ &= \int_{\mathbb{R}} d\sigma \delta q(\sigma) \int_{\mathbb{R}} dt \frac{\delta L(T_t q, t)}{\delta q(\sigma)} = 0 \end{aligned}$$